Thursday, May 19, 2016 5:20 PM

Evaluate $\int_C y^2 dx + x dy$

i) $C = C_1$ is the line segment from $(0,2)$ to $(-5,-3)$

ii) $C = C_2$ is the arc of the parabola $x = 4-y^2$ from $(0,2)$ to $(-5,-3)$

Soln

i) The parametrization of $C = C_1$ is given by

$$\vec{r}(t) = (1-t)\langle 0,2\rangle + t\langle -5,-3\rangle \quad , \; 0 \le t \le 1$$
$$= \langle 0, 2-2t\rangle + \langle -5t, -3t\rangle$$
$$= \langle -5t, 2-5t\rangle$$

So, $x = -5t$, $y = 2-5t$, $0 \le t \le 1$.

$$\int_C y^2 dx + x dy = \int_0^1 (2-5t)^2 . -5dt + -5t . -5dt$$
$$= -5\int_0^1 4 - 25t + 25t^2 \, dt = -5\left[4t - \frac{25t^2}{2} + \frac{25t^3}{3}\right]_0^1 = -5\left[4 - \frac{25}{2} + \frac{25}{3}\right] = \frac{5}{6}$$

ii) Since the parabola is given as a function of y, we can take y as a parameter. [Or set $y=t$ and you get the same answ]

Then,

$x = 4-y^2$, $y = y$, where y goes from 2 to -3

Then $dx = -2y\,dy$ and

$$\int_{C_2} y^2 dx + x dy = \int_2^{-3} y^2(-2y)dy + (4-y^2)\,dy = \int_2^{-3} (-2y^3 - y^2 + 4)\,dy = \left[-\frac{y^4}{2} - \frac{y^3}{3} + 4y\right]_2^{-3}$$
$$= \left[\frac{-(-3)^4}{2} - \frac{(-3)^3}{3} - 12\right] - \left[-\frac{(+2)^4}{2} - \frac{2^3}{3} + 8\right]$$
$$= \frac{-81}{2} + 9 - 12 - 8 + 8 + \frac{8}{3} = \frac{-243 - 18 + 16}{6} = -\frac{245}{6}$$

$\boxtimes$